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Ply Angle Optimization of Nonuniform Composite Beams Subject to Aeroelastic Constraints

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I. Introduction

SEVERAL studies on the optimization of aeroelastically constrained, composite wings with cantilever end conditions have been conducted. For example, recent optimizations[1,2] have examined the design of nonuniform, flat composite beams for frequency, flutter, and divergence constraints. During these studies, variation of the thickness of a generic layup with fixed values of ply angles was considered, and an experimental validation was carried out. Other related work[3–6] has investigated the influence that ply orientation, sweep angle, wash-in, and wash-out, as well as various other parameters, has on flutter speed. This work showed that modal interchange can significantly alter the flutter speed of a composite wing[4] and that for flutter of an unswept composite wing, torsional rigidity and coupled bending–torsional rigidity[5] are the most influential parameters. It was also found that, contrary to traditional thinking, wash-out can be beneficial from a flutter point of view.[6] A recent independent study[7] has examined the effect of both ply angle variation and the position of lumped masses on flutter speed for uniform thickness wind-tunnel models. Here, the principal findings were that small variations in thickness can have a significant effect on flutter speed and that practical application of optimization should allow for uncertainties in the aerodynamic and structural models.

For all of the cited studies in which ply angle has been varied, the layup has been constructed from either unidirectional material [0 deg] or woven $[0/90]_S$ material. Also, the wing structure has previously consisted of a flat composite beam of uniform thickness. The current paper presents results in two parts, both of which are based on the model that has previously been optimized[1,2] and, in one case, experimentally tested.[1] The first part examines the effect that varying the orientation α of a $[90+\alpha/0+\alpha/+45+\alpha/-45+\alpha]_S$ layup has on the flutter and divergence speeds of a uniform beam that is both unswept and swept back, where the effect of such variation is to alter the influence of each layer on the beam rigidities. In the second part, the design optimization of nonuniform beams with varying orientation of the same layup is considered. The wing model consists of 10 uniform beam elements, where each element has a layup of $[90+\alpha/0+\alpha/+45+\alpha/-45+\alpha]_S$, a length of 0.04 m, and a width of 0.08 m (Fig. 1). The layup orientation α, which was previously[1,2] always 0 deg, is defined as the angle that the 0-deg fibers are inclined to the y axis of the beam. (Note that for a swept wing the y axis is the centroidal axis of the beam.) The structural beam is enclosed in a NACA 0015 airfoil of (unswept) chord 0.195 m. The mid-chord position of this airfoil is positioned 0.04 m in front of the beam center.

Analysis and optimization are described in detail in Ref. 2. Briefly, the dynamic stiffness method (DSM) is used to carry out free vibration analysis for the nonuniform composite beam by idealizing it as a series of uniform beam elements with bending, torsional, and coupled bending–torsional rigidities, EI, GJ, and K, respectively, where positive K causes the wing to twist leading edge down when it is bent upward. The effect of shear deformation, which is ignored in the model, is relatively small because DSM frequencies are relatively close to experimental and finite element results that allow for shear deformation.[1] The flutter speed V_f is found using the V–g method and the divergence speed V_d is obtained by considering it as a static (zero-frequency) instability problem.

II. Results

A. Parametric Study for Uniform Thickness Beam

The analysis of a uniform thickness beam model for varying layup orientation α, where each of the eight layers within the $[90+\alpha/0+\alpha/+45+\alpha/-45+\alpha]_S$ layup has the same thickness, is first considered. Figure 2 shows the variation of flutter and divergence speed with varying α for an unswept wing. (Here, as in Fig. 3, airspeed has been normalized against the flutter speed found when $\alpha = 0$ deg.) The maximum flutter speed occurs at around $\alpha = 52$ deg,

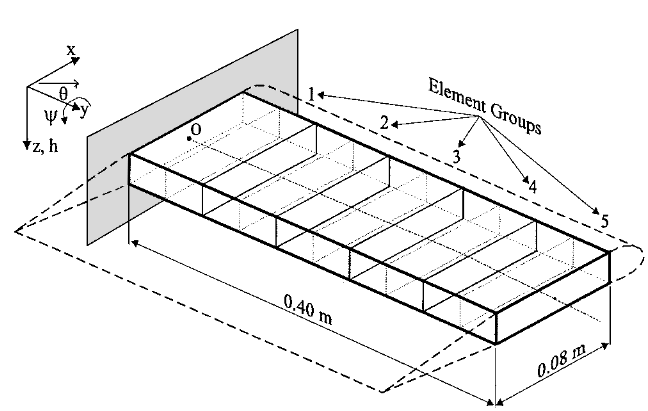

Fig. 1 Unswept wing model with coordinate system, where θ shows the direction of both positive layup orientation and positive ply angle.

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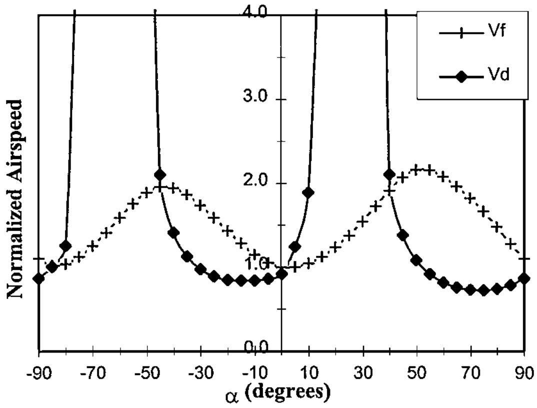

Fig. 2 Flutter and divergence speed against layup orientation α for unswept wing.

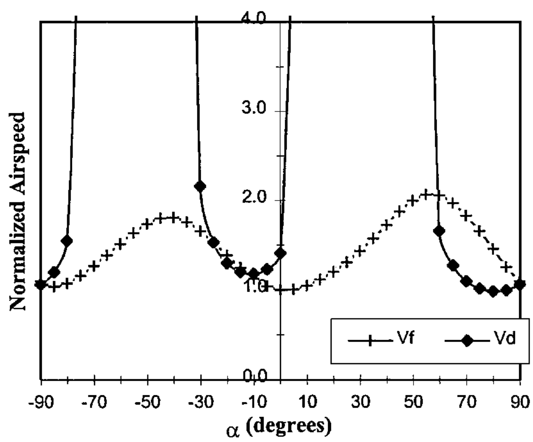

Fig. 3 Flutter and divergence speed against layup orientation α for wing with 20-deg sweep.

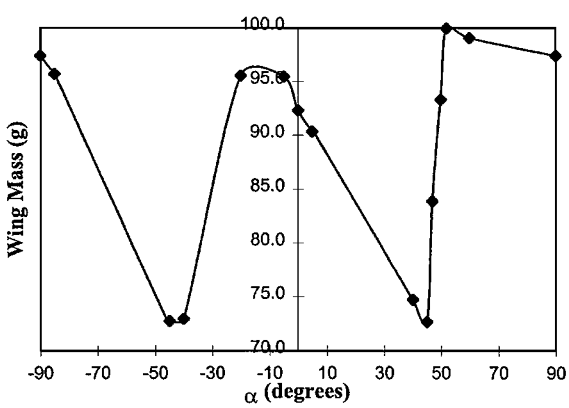

Fig. 4 Optimum mass against layup orientation α.

which corresponds to a large value of GJ, low EI, and a small amount of negative K. It is also evident that divergence speed approaches infinity for maximum values of positive K. Peak values on the flutter and divergence envelope shown in Fig. 2 occur at α values of around +41 and −45 deg.

When the wing model is swept back 20 deg, the flutter and divergence results of Fig. 3 are obtained. As expected, the regions in which divergence is critical are reduced compared with the unswept case. These regions, which have low values of GJ and negative K, are confined to the range $\alpha = 60$–90 deg and around $\alpha = -20$ deg. The flutter/divergence boundary maximum now occurs when flutter speed reaches a maximum, that is, at around $\alpha = 60$ deg. This corresponds to relatively high values of both GJ and negative K.

B. Optimization of Nonuniform Thickness Beam

During optimization, the beam is divided into five element groups of length and width 0.08 m (see Fig. 1). The thickness of each of the (90 deg + α), (0 deg + α), (45 deg + α), and (−45 deg + α) layers within each group is allowed to vary within the limits of 0.125 and 2.0 mm. To find a minimum-mass wing design that satisfies aeroelastic constraints, a sequential quadratic programming strategy combined with the modified methods of feasible directions optimizer contained in the optimization package DOT[8] is used. The minimum acceptable flutter and divergence speeds for this beam are set to 36.91 and 37.91 m/s, respectively, these being the values used in the previous study[2] in which layup orientation was not varied.

Initially, the unswept beam was optimized for a range of α values (Fig. 4). Each point in Fig. 4 gives the optimum mass produced by a single optimization run for a single α value, taking on average 4 CPU hours on an Silicon Graphics O_2 (175-MHz/R10,000) workstation. The relationship between Fig. 4 and the flutter/divergence boundary of Fig. 2 is clearly apparent. For instance, the optimum (minimum-mass) values of α in Fig. 4, occurring at around +42 and −45 deg correspond to maximum points in Fig. 2. These optimum designs show a 23% reduction in mass compared to the $\alpha = 0$ case.

Table 1 Optimum designs for unswept ($\Omega = 0$ deg) and swept ($\Omega = 20$ deg) wings with different values of layup orientation α, deg

Property	$\Omega = 0$, $\alpha = 0$	$\Omega = 0$, $\alpha = 45$	$\Omega = 20$, $\alpha = 60$
V_f, m/s	36.91	37.10	36.91
V_d, m/s	37.94	37.89	37.89
Element group			
	(90 deg + α) layer thickness, mm		
1	0.125	0.125	0.125
2	0.125	0.125	0.134
3	0.125	0.125	0.204
4	0.125	0.125	0.198
5	0.125	0.125	0.125
	(0 deg + α) layer thickness, mm		
1	0.125	0.432	0.230
2	0.125	0.411	0.263
3	0.125	0.374	0.262
4	0.125	0.309	0.203
5	0.125	0.132	0.125
	(45 deg + α) layer thickness, mm		
1	0.271	0.183	0.285
2	0.292	0.173	0.244
3	0.287	0.144	0.183
4	0.248	0.126	0.137
5	0.160	0.125	0.125
	(−45 deg + α) layer thickness, mm		
1	0.548	0.126	0.237
2	0.507	0.126	0.210
3	0.452	0.126	0.171
4	0.383	0.126	0.137
5	0.254	0.125	0.125
	Rigidity EI, Nm^2		
1	3.90	0.76	1.45
2	3.70	0.68	1.36
3	3.14	0.53	1.48
4	2.27	0.38	0.89
5	0.95	0.19	0.35
	Rigidity GJ, Nm^2		
1	5.51	5.47	4.07
2	5.06	4.93	3.77
3	4.04	3.90	3.65
4	2.62	2.77	2.08
5	0.81	1.01	0.82
	Rigidity K, Nm^2		
1	0.69	0.17	−0.98
2	0.78	0.14	−1.06
3	0.70	0.07	−1.46
4	0.47	0.02	−0.91
5	0.14	−0.06	−0.32
Total mass, g	92.3	72.7	73.8

Furthermore, values of α that produce high minimum-mass designs (α values of around −20 and 75 deg) correspond to minimum points on the flutter and divergence boundary of Fig. 2.

The $\alpha = 0$ deg optimum design for zero sweep ($\Omega = 0$ deg), which was obtained previously,[2] and the $\alpha = 45$-deg optimum design are compared in Table 1. It can be seen that the (−45 deg + α) layer is thickest for the $\alpha = 0$ deg optimum, whereas for the $\alpha = 45$ deg optimum, the (0 deg + α) layer is thickest. In the latter case, the (0 deg + α) layer is torsionally more efficient than the $\alpha = 0$ deg, (−45 deg + α) layer, as a result of being located farther from the center of the beam. Hence, although the torsional rigidity of both designs is similar, the $\alpha = 45$ deg mass is considerably lower, and as a result of the position of 0-deg fibers in the $\alpha = 0$ deg design, its bending rigidity is high when compared with the $\alpha = 45$ deg design. This will result in reduced separation between first bending and first torsional natural frequencies giving reduced flutter speed. Hence, the required minimum flutter speed is achieved for the $\alpha = 0$ deg case by a comparatively large value of positive coupled bending–torsional rigidity, which causes the wing to wash-out. This effect reduces the coupling between aerodynamic forces (acting at quarter chord) and the torsional center of the wing.

Table 1 shows that for the $\alpha = 0$ deg optimum both the (90 deg + α) and (0 deg + α) layers are at their lower bound. Although this bound may be thought of as representing the practical constraints of strength and layup stability, which are not considered here, an additional optimization with a lower bound of 0.001 mm was performed to investigate the effect of allowing the optimizer to remove this material altogether. This gave an optimum mass of 81.39 g, which is still around 10% heavier than the $\alpha = 45$ deg optimum. Hence, the advantage associated with the inclusion of α as a design variable would appear to be confirmed.

A similar study, performed for the wing with 20-deg sweep, also confirmed that the optimum value of α corresponds to the α producing maximum airspeed values on the flutter/divergence boundaries of Fig. 3. Hence, optimization results for the optimum orientation, $\alpha = 60$ deg, are given in Table 1 with the associated rigidities. It can be seen that the optimum design in this case has negative coupled bending–torsional rigidity, causing the wing to wash-in.

To check the sensitivity of the $\Omega = 20$ deg, $\alpha = 60$ deg optimum design to small changes in α that may arise from manufacturing imperfections, the optimum design was reanalyzed with α values of 55 and 65 deg. This analysis gave, respectively, flutter speeds of 36.76 and 35.39 m/s and divergence speeds of 114.88 and 25.35 m/s. Hence, the divergence speed of the optimum is very sensitive to imperfection, which can be explained by the very steep variation in divergence speed at α values of around 60 deg in Fig. 3.

III. Conclusions

The results described show the variation of flutter and divergence speeds of a uniform composite wing with layup $[90 + \alpha/0 + \alpha/+45 + \alpha/-45 + \alpha]_S$ when subject to variations in layup orientation α. Following this, optimum nonuniform beam designs have been obtained for both swept and unswept wings for varying values of α. At the conceptual structural design stage, a wing designer cannot afford to run a series of optimization runs to cover the full range of α between +90 and −90 deg to find the optimum α. Hence, it is anticipated that the proposed strategy, in which the layup orientation giving maximum airspeed on the flutter/divergence boundary for a uniform wing is found to correspond to the optimum α for nonuniform wings, would be very advantageous. Furthermore, this method of finding the optimum α typically takes only 5% of the CPU time required for a single optimization run.

It is obviously important to consider the practicality of the preceding optimization results, particularly from a manufacturing point of view. It has been shown that the optimum designs may be sensitive to small changes (imperfections) in α. Furthermore, any continuous optima would need to be converted to a discrete number of layers as has been done previously.[2] This may be difficult, if not impossible, considering the small range of thicknesses in some cases. However, the aim of the paper is to show the potential benefit associated with design involving variable layup orientation.

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Stability of Orthotropic Plates on a Kerr Foundation

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Nomenclature

A_{mn}	= undetermined coefficients
a, b	= sides of the plate
c	= aspect ratio, identical to a/b
D_1, D_2	= flexural rigidities in principal directions
D_3	= $D_1 \nu_2 + G'h^3/6$
E_1, E_2	= Young's moduli for the principal directions
G	= shear modulus for the shear layer
G'	= modulus of rigidity of orthotropic plate
k_1	= foundation modulus of the upper spring layer
k_2	= foundation modulus of the lower spring layer
m	= half-sine wave number in x direction (integer)
n	= half-sine wave number in y direction (integer)
$\bar{P}$	= nondimensional critical load, identical to $pb^2/\{\pi^2\sqrt{(D_1D_2)}\}$
$\bar{P}_{\min}$	= minimum nondimensional critical load
p_{cr}	= critical load
p_x	= in-plane load in x direction
p_y	= in-plane load in y direction
p_1	= contact pressure under the plate
p_2	= contact pressure under the shear layer
w	= plate deflection
w_1	= deflection of the upper spring layer
w_2	= deflection of the shear layer
α	= coefficient
λ_1	= nondimensional foundation modulus of the upper spring layer, identical to $k_1b^4/\{\pi^4\sqrt{(D_1D_2)}\}$
λ_2	= nondimensional foundation modulus of the lower spring layer, identical to $k_2b^4/\{\pi^4\sqrt{(D_1D_2)}\}$

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